(7 pages)

S.No. 370

17PPH02

(For the candidates admitted from 2017–2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

First Semester

Physics

MATHEMATICAL PHYSICS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 1 = 10 marks)

Answer ALL questions.

Choose the correct answer.

1. Bessel's inequality is written as

(a) $\int_a^b |f(x)|^2 W(x)\,dx \geq \sum_{n=0}^{\infty} |C_n|^2$

(b) $\int_a^b |f(x)|^2 W(x)\,dx \leq \sum_{n=0}^{\infty} |C_n|^2$

(c) $\int_a^b |f(x)|^2\,dx \geq \sum_{n=0}^{\infty} |C_n|^2$

(d) $\int_a^b |f(x)|^2\,dx \leq \sum_{n=0}^{\infty} |C_n|^2$

2. In the case of orthogonal curvilinear coordinates,

(a) $\hat{u}_i \times \hat{u}_i = \hat{u}_k$ (b) $\hat{u}_i \cdot \hat{u}_i = 1$

(c) $\hat{u}_i \cdot \hat{u}_j = 1$ (d) $\hat{u}_i \times \hat{u}_j = 0$

3. All diagonal elements of a Hermitian matrix mare

(a) zero

(b) pure imaginary numbers

(c) real numbers

(d) either zero or pure imaginary numbers

4. Maximum number of independent components of a symmetric tensor of rank 2 in n-dimensional space is

(a) $\frac{n}{2}$ (b) $\frac{(n+1)}{2}$

(c) $\frac{n^2}{2}$ (d) $\frac{n(n+1)}{2}$

5. Complex form of Fourier series is expressed as

(a) $f(x) = \sum_{n=-\infty}^{\infty} C_n e^{inx}$ (b) $f(x) = \sum_{n=0}^{\infty} C_n e^{-inx}$

(c) $f(x) = \sum_{n=0}^{\infty} C_n e^{inx}$ (d) $f(x) = \sum_{n=1}^{\infty} C_n e^{-inx}$

6. Laplace transform of 't' is

(a) $1/s$ (b) $1/s^2$

(c) $2/s^3$ (d) 0

7. The value of $\int_0^{\pi i} z \cos z^2 \, dz$ is

(a) ∞ (b) $1/2$

(c) $-1/2 \sin \pi^2$ (d) 1

8. Cauchy-Riemann equations are

(a) $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}; \frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$

(b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}; \frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$

(c) $\frac{\partial u}{\partial x} = -\frac{\partial v}{\partial y}; \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

(d) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}; \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$

9. If $P_n(x)$ denotes the Legendre's polynomials, then $P_{2m+1}(0) =$ ————————.

(a) 0 (b) 1

(c) x^{2m} (d) x^{2m+1}

10. Which of the following represents Laplace's equation?

(a) $\nabla^2 V = \frac{\rho}{\varepsilon_0}$ (b) $\nabla^2 V = 0$

(c) $\nabla^2 V = \frac{\partial V}{\partial t}$ (d) $\nabla^2 V = \frac{\partial^2 V}{\partial t^2}$

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Define a Hilbert space and prove that square integrable functions form a Hilbert space.

Or

(b) What are linear operators? Prove that the operator $\hat{R}_z$ which rotates every vector in the vector space V_3 by an angle 'θ' in the anti-clockwise direction about the z axis is linear.

12. (a) What is known as similarity transformation? Show that the trace of a matrix remains invariant under this transformation.

Or

(b) Define the Kronecker delta symbol? And discuss its properties.

13. (a) Expand as a Fourier series the function $f(x) = x^2$ in the interval $-\pi < x < \pi$ and hence evaluate $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Or

(b) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

14. (a) State and prove the Cauchy's integral formula.

Or

(b) Evaluate the residue of $f(z) = \frac{z^3}{(z-1)^4 (z-2)(z-3)}$ at its simple poles.

15. (a) Show that $x J_n'(x) = n J_n(x) - x J_{n+1}(x)$.

Or

(b) State and prove the orthogonal property of Laguerre polynomials.

SECTION C — (5 × 8 = 40 marks)

Answer ALL questions.

16. (a) Explain the Schmidt's orthogonalization method to obtain a set of orthonormal vectors.

Or

(b) Obtain the equation for 'divergence' in orthogonal curvilinear coordinates.

17. (a) Obtain the eigen values and normalised eigen vectors of $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.

Or

(b) (i) Define the inner product of two tensors.

(ii) State Quotient law in tensor analysis and explain it with an example.

18. (a) (i) Find the Fourier integral of the function $f(x) = e^{-kx}$ when $x > 0$ and $f(-x) = f(x)$, $k > 0$.

(ii) Hence deduce that $\int_0^{\infty} \frac{\cos xu}{1+u^2} du = \frac{\pi}{2} e^{-x}$ $(x > 0)$.

Or

(b) (i) Find the inverse Laplace transform of $\frac{3}{s^2+9}$.

(ii) Using Laplace transformation method, solve the differential equation $\frac{d^2y}{dx^2}+9y=0$ with $y(0)=0$ and $\left.\frac{dy}{dx}\right|_{x=0}=0$.

19. (a) State and prove the Taylor's series expansion of an analytic function $f(z)$ with centre at z_0.

Or

(b) Evaluate by contour integration $\int_0^\infty \frac{\sin \pi x}{x(1-x^2)}dx$.

20. (a) Find the power series solution of linear oscillator equation $\frac{d^2y}{dx^2}+w^2y=0$ in powers of x (that is, near $x=0$).

Or

(b) Prove that :

(i) $H'_n(x)=2nH_{n-1}(x)$

(ii) $2xH_n(x)=2nH_{n-1}(x)+H_{n+1}(x)$.