(b) (i) Prove the orthogonality relation $\int_{-1}^{1} P_n(x) P_m(x) dx = 0$ if $m \neq n$.

(ii) Show that $\frac{e^{-xt/1-t}}{1-t} = \Sigma L_n(x) t^n$.

10. (a) (i) Derive relation between beta and gamma function.

(ii) Show that

$$\int_0^{\pi/2} \cos^p \theta \sin^q \theta \, d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$$

Or

(b) (i) Write short note on dirac delta function and its properties.

(ii) For a positive integer n, show that

$$\Gamma(1/n)\,\Gamma(2/n)\ldots\Gamma[(n-1)/n] = \frac{(2\pi)^{(n-1)/2}}{\sqrt{\pi}}.$$

(For the candidates admitted from 2012-2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

First Semester

Physics

MATHEMATICAL PHYSICS

Time : Three hours Maximum : 75 marks

PART A — (5 × 5 = 25 marks)

Answer ALL questions.

1. (a) Show that $curl(grad\,\psi) = 0$ in terms of orthogonal curvilinear co-ordinates.

Or

(b) State and prove Gauss divergence theorem.

2. (a) Find the Fourier transform of function $f(t) = e^{-|t|}$.

Or

(b) Find the inverse Laplace transform of the function $F(s) = \frac{s}{(s+a)(s+b)}$ $a \neq b$.

3. (a) Determine the analytic function $f(z) = u + iv$ whose imaginary part is $v = 6xy - 5x + 3$.

Or

(b) Find Poles and residues at the poles of the function $f(z) = \dfrac{e^z}{z^2 + a^2}$.

4. (a) Derive the Rodrigue's formula for the Hermite polynomial.

Or

(b) Show that

$$(2m+1)x\,P_m(x) = (m+1)P_{m+1}(x) + (m-1)P_{m-2}(x)$$

5. (a) Define Beta function $\beta(m,n)$, show that it is symmetric about its indices m and n.

Or

(b) Show that $\Gamma(m)\,\Gamma(1-m) = \dfrac{\pi}{\sin m\pi}$.

PART B — (5 × 10 = 50 marks)

Answer ALL questions.

6. (a) Derive the expressions for (i) gradient of a scalar field (ii) divergence of a vector field and (iii) Laplacian operator in orthogonal Curvilinear coordinates.

Or

(b) State and prove Stoke's theorem and deduce the following relation $\iint ds \times \nabla\phi = \oint \phi\, dl$.

7. (a) Find Fourier sine and cosine transform of $f(t) = e^{-pt}$ $p > 0$. Hence evaluate $\int_0^\infty \dfrac{\cos wt}{p^2 + w^2}\,dw$ and $\int_0^\infty \dfrac{w \sin wt}{p^2 + w^2}\,dw$.

Or

(b) Solve the differential equation using Laplace transform, $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + 5y = e^{-x}\sin x$ where $y(0) = 0$ and $y'(0) = 1$.

8. (a) State and prove Laurent series for complex variables.

Or

(b) With the help of the calculus of residues, evaluate integral $\int_{-\infty}^{\infty} \dfrac{\cos(px) - \cos(qx)}{x^2}\,dx$.

9. (a) Find out the solution of Bessel differential equation $x^2\dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (x^2 - n^2)y = 0$ where n is an integer.

Or