SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that two solutions ϕ_1, ϕ_2 of $L(y) = y'' + a_1 y' + a_2 y = 0$ are linearly independent on an interval I if and only if, $W(\phi_1, \phi_2)(x) \neq 0$.

17. Find all solutions of $y'' + y = \sec x$, $\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right)$.

18. Prove that there exists n linearly independent solutions of

$$L(y) = y^n + a_1(x) y^{(n-1)} + \ldots + a_n(x) y = 0 \text{ on } I.$$

19. Show that $\int_{-1}^{1} P_n(x) P_m(x) dx = \begin{cases} 0, & if\ n \neq m \\ \frac{2}{2n+1}, & if\ n = m \end{cases}$.

20. State and prove Existence theorem.

S.No. 362 **17PMA04**

(For the candidates admitted from 2017–2018 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Solve $y'' + \omega^2 y = 0$ where ω is a positive constant.

2. Write down the formula for wronskian.

3. State initial value problem for n-th order equation.

4. State the existence theorem for n-th order equation.

5. State the uniqueness theorem for n-th order equation.

6. Define wronskian and linearly independence for n-th order equation.

7. Write down the solution of Bessel's equation.

8. Write down the second order equation with regular singular point.

9. Solve $y' = 3y^{2/3}$ by the method of variable separable.

10. Write down the Lipschitz condition.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Discuss Linearly dependence and independence.

Or

(b) Let ϕ_1 and ϕ_2 be any two linearly independent solutions of $L(y) = y'' + a_1 y' + a_2 y = 0$ on an interval I. Prove that every solution ϕ can be written uniquely as $\phi = c_1\phi_1 + c_2\phi_2$, where c_1 and c_2 are constants.

12. (a) Find all the solutions of $y^{(4)} + 16y = 0$.

Or

(b) Use annihilator method find a particular solution of $y'' + 4y = \sin 2x$.

13. (a) Discuss the solution of the homogeneous equation of n-th order.

Or

(b) Find the linearly independent solutions of the equation

$$(3x-1)^2 y'' + (9x-3)y' - 9y = 0 \text{ for } x > \frac{1}{3}.$$

14. (a) Show that $K_0'(x) = -K_1(x)$.

Or

(b) Show that $J_{\alpha-1}(x) + J_{\alpha+1}(x) = 2\alpha x^{-1} J_\alpha(x)$.

15. (a) Let M, N be two real valued functions which have continuous first partial derivatives on some rectangle $R: |x - x_0| \leqq a$, $|y - y_0| \leqq b$. Prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Or

(b) Compute the first four successive approximations $\phi_0, \phi_1, \phi_2, \phi_3$ of the equation $y' = y^2$, $y(0) = 1$.