(6 pages)

S.No. 229

12PMAZ01

(For the candidates admitted from 2012 – 2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

First Semester

Mathematics

NUMERICAL ANALYSIS

Time : Three hours   Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL the questions.

1. Write down Taylor's formula.

2. Write Milne's corrector formula.

3. Write the general formula for Euler's method.

4. Write the n^{th} approximation of $\frac{dy}{dx} = f(x,y)$ such that $y(x_0) = y_0$ by Picards method.

5. Write R-K- second order formula to solve $y' = f(x,y),\ y(x_0) = y_0$.

6. Write R-K fourth order formula to solve $y' = f(x,y), y(x_0) = y_0$.

7. State the standard five point formula for solving $u_{xx} + u_{yy} = 0$.

8. When you say the second order partial differential equation $A(x,y)\frac{\partial^2 u}{\partial x^2} + B(x,y)\frac{\partial^2 u}{\partial x \partial y} + c(x,y)\frac{\partial^2 u}{\partial y^2} + f\left(x, y, u, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}\right) = 0$ to be elliptic.

9. Write down the one dimensional heat equation.

10. State Bendre – Schmidt recurrence formula.

PART B — (5 × 5 = 25 marks)

Answer ALL the questions.

11. (a) Using Taylor series method, find $y(0.1)$ given $\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$.

Or

(b) Evaluate the solution to the differential equation $y' = (1+x)xy^2$ subject to $y(0) = 1$ by taking five terms in Maclaurin's series for $x = 0(0.1)0.4$.

12. (a) Use Picards method to approximate the value of y when $x = 0.1$ given that $y = 1$ at $x = 0$ and $y = 1 + xy$, correct to three decimal places.

Or

(b) Solve $\frac{dy}{dx} = 1 - y,\ y(0) = 0$ in the range $0 \le x \le 0.3$ using Euler's method.

13. (a) Given $y' = x^2 - y,\ y(0) = 1$, find $y(0.1)$ using Runge Kutta method of third order.

Or

(b) Using Runge – Kutta method of fourth order solve for $y(0.1)$ given that $y' = xy + y^2$, $y(0) = 1$.

14. (a) Discuss the solution to Laplace's equation by Liebmann's iteration process.

Or

(b) Classify :

(i) $3u_{xx} + u_{xy} - 4u_{yy} + 3u_y = 0$

(ii) $x^2 u_{xx} + (a^2 - y^2)u_{yy} = 0;$

$-\infty < x < \infty, -a < y < a.$

15. (a) Find the solution to $u_t = u_{xx}$, subject to $u(x,0) = \sin \pi x,\ 0 \le x \le 1, u(0, t) = u(1,t) = 0$ using Schmidt method.

Or

(b) Solve $u_{tt} = 4u_{xx}$ with the boundary conditions $u(0,t) = 0 = u(4,t), u_t(x,0) = 0$ and $u(x,0) = x(4-x)$.

PART C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Using Taylors series method, solve $\frac{dy}{dx} = xy + y^2, y(0) = 1$ at $x = 0.1, 0.2$ and 0.3 continue the solution at $x = 0.4$ by Milne's Predictor corrector method.

17. Solve $\frac{dy}{dx} = y - \frac{2x}{y}, y(0) = 1$ in the range $0 \leq x \leq 0.2$ using

(a) Euler's method

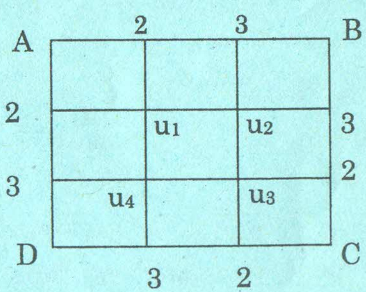
(b) Improved Euler's method

(c) Modified Euler's method.

Take $h = 0.1$

18. Solve $\frac{dy}{dx} = yz + x; \frac{dz}{dx} = xz + y$ given that $y(0) = 1; z(0) = -1$ for $y(0.2), z(0.2)$.

19. Solve $u_{xx} + u_{yy} = 0$ for the following square mesh with boundary values as shown in figure. Iterate until the maximum difference between the successive values at any point is less than 0.001.



20. Solve the equation $\nabla^2 u = -10(x^2 + y^2 + 10)$ over the square mesh with sides $x = 0, y = 0, x = 3, y = 3$ with $u = 0$ on the boundary and mesh length 1.