17. Prove that A graph G with $\upsilon \geq 3$ is 2-connected if and only if any vertices of G are connected by at least two internally-disjoint paths.

18. Prove that every 3-regular graph without cut edges has a perfect matching.

19. State and prove Vizing's theorem.

20. State and prove Brooks theorem.

(For the candidates admitted from 2012 – 2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Fourth Semester

Mathematics

GRAPH THEORY

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define simple graph.

2. Define bipartite graph.

3. Define tree.

4. Explain cut vertex.

5. Define Eulerian graph.

6. Define matching.

7. Explain edge chromatic number.

8. Define clique.

9. Define vertex colouring.

10. Explain Hajo's conjecture.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that $\sum_{u \in V} d(v) = 2 \in$.

Or

(b) Explain Dijkstra's algorithm.

12. (a) Prove that a connected graph is a tree if and only if every edge is a cut edge.

Or

(b) If e is a link of G then prove that $\tau(G) = \tau(G-e) + \tau(G \cdot e)$.

13. (a) If G is hamiltonian then prove that for every non empty proper subset S of V, $w(G-S) \leq |S|$.

Or

(b) If G is a K-regular bipartite graph with $K > 0$, then G has a prefect matching.

14. (a) State and prove Ramsey's theorem.

Or

(b) Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge colouring in which both colours are represented at each vertex of degree atleast two.

15. (a) Prove that every critical path is a block.

Or

(b) If G is simple, then prove that $\pi_k(G) = \pi_k(G-e) - \pi_k(G.e)$ for any edge e of G.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Explain :

(a) incident matrix

(b) adjacency matrix

(c) path

(d) connected graph

(e) cycle.