SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Derive the Euler's equation of the variation problem.

17. Find the extremum of the functional

$$I=\int_{x_1}^{x_2}\left(y'^2+z'^2+2\,yz\right)dx \text{ with } y(0)=0, z(0)=0 \text{ and}$$

the point (x_2, y_2, z_2) moves over the fixed plane $x=x_2$.

18. Transform $\frac{d^2y}{dx^2}+xy=1, y(0)=y(1)=0$ into an integral equation.

19. Derive the solution of Fredholm integral equation of the second kind using Separable kernel method.

20. Solve the symmetric integral equation by using Hilbert-Schmidt theorem $y(x)=(x+1)^2+\int_{-1}^{1}\left(xt+x^2\,t^2\right)y(t)\,dt$.

————————

12PMA11

(For the candidates admitted from 2012–2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Third Semester

Mathematics

CALCULUS OF VARIATION AND INTEGRAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Write the Euler-Poisson equation.
2. State Hamilton's principle.
3. State Fermat's principle.
4. State orthogonality condition for moving boundary.
5. Define Eigen function.
6. Define initial value problem.
7. Write the Volterra equation of the second kind.
8. Write the separable kernel of Fredholm integral equation.

9. When we say the orthonormal system of function is normalized.

10. Define Hilbert space.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Test for an extremum the functional $I[y(x)] = \int_0^{\frac{\pi}{2}} (y'^2 - y^2)\, dx \quad y(0) = 0,\ y\left(\frac{\pi}{2}\right) = 1$.

Or

(b) Derive the Euler-Ostrogradsky equation for variation problem.

12. (a) Using only the basic necessary condition $\delta I = 0$ find the curve on which an extremum of the functional $I[y(x)] = \int_0^{x_1} \frac{(1 + y'^2)^{1/2}}{y} dx$, $y(0) = 0$ can be achieved if the second boundary point (x_1, y_1) can move along the circumference $(x-9)^2 + y^2 = 9$.

Or

(b) Derive the Weirstrass-Erdmann corner conditions.

13. (a) Convert the following initial value problem into an integral equation : $y'' + y = 0$ when $y(0) = y'(0) = 0$.

Or

(b) Write the types of Kernel.

14. (a) Solve: $y(x) = x + \int_0^{1/2} y(t)\, dt$.

Or

(b) Show that the homogeneous integral equation $y(x) = \lambda(3x - 2) \int_0^1 t\, y(t) dt$ has no characteristic numbers and eigen functions.

15. (a) State and prove Hilbert – Schmidt theorem.

Or

(b) Explain Gram-Schmidt orthogonolization process.