18. If f is absolutely continuous on $[a, b]$ and $f'(x) = 0$ almost everywhere then prove that f is constant.

19. State and prove Hahn decomposition theorem.

20. State and prove Fubini theorem.

S.No. 240

12PMA10

(For the candidates admitted from 2012 – 2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Third Semester

Mathematics

MEASURE THEORY AND INTEGRATION

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define outer measure of a set.

2. When do we say a function is Lebesgue measurable?

3. Define Simple function.

4. State Monotone convergence theorem.

5. State Vitali lemma.

6. Define Singular.

7. Define complete.

8. State Radon-Nikodym theorem.

9. Define semi algebra.

10. State Tonelli theorem.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If $\{A_n\}$ is a countable collection of sets of real numbers, the prove that $m^*(\bigcup A_n) \le \sum m^* A_n$.

Or

(b) If f is a measurable function and $f = g$ almost everywhere, then prove that g is measurable.

12. (a) State and prove Fatou's lemma.

Or

(b) State and prove Lebesgue convergence theorem.

13. (a) If f is integrable on [a, b], then prove that the function F defined by $F(x) = \int_a^x f(t)dt$ is a continuous function of bounded variation on [a, b].

Or

(b) If f is absolutely continuous on $[a, b]$, then prove that it is of bounded variation $[a. b]$.

14. (a) If $E_1 \, \varepsilon \, \mathbf{B}$, $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n \to \infty} \mu E_n$.

Or

(b) State and prove Lebesgue decomposition theorem.

15. (a) If $A \varepsilon \, a$ then show that A is measurable with respect to μ^*.

Or

(b) If x be a point of X and E a set in $\mathcal{R}_{\sigma\delta}$ then prove that E_x is a measurable subset of Y.

SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Prove that the outer measure of an interval is its length.

17. State and prove bounded convergence theorem.