18. State and prove Intermediate value theorem.

19. State and prove the uniform continuity theorem.

20. State and prove the Urysohn lemma.

S.No. 239 **12PMA09**

(For the candidates admitted from 2012 – 2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Third Semester

Mathematics

TOPOLOGY

Time : Three hours Maximum : 75 marks

PART A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define discrete topology.

2. Define basis for a topology.

3. Define metric space.

4. Define uniform metric.

5. Define linear continuum.

6. Define the Cartesian product

7. State the tube lemma.

8. Define limit point compact.

9. Define Hausdorff space.

10. Define completely regular.

PART B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Let X be a topological space. Then prove that the following conditions hold:

(i) ϕ and X are closed

(ii) Arbitrary intersections of closed sets are closed.

Or

(b) Show that every finite point set in a Hausdorff space is closed.

12. (a) Show that the box topology in the product space $\prod X_\alpha$ is finer than the product topology.

Or

(b) State and prove the Pasting lemma.

13. (a) If A is a connected subset of X, then prove that any set B such that $A \subset B \subset \overline{A}$ is also connected.

Or

(b) Prove that a spaces X is connected if and only if X and the empty set are the only both closed and open sets.

14. (a) Show that every closed subset of a compact space is compact.

Or

(b) State and prove Extreme value theorem.

15. (a) Show that the product of two Lindelof space need not be Lindelof.

Or

(b) Show that every compact Hausdorif space is normal.

PART C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If X and Y are two topological spaces and Π_1 and Π_2 are the projection of $X \times Y$ onto X and Y respectively, then prove that the collection

$$S = \{\Pi_1^{-1}(U) / U \text{is open in} X\} \cup \{\Pi_1^{-1}(V) / V \text{ is open in } Y\}$$

is a sub basis for a product topology.

17. Let X and Y be topological spaces. Let $f : X \times Y$ then show that the following are equivalent

(a) f is continuous

(b) For every subset A of X.

(c) For every closed set in Y, then the set $f^{-1}(B)$ is closed in X.