(b) Derive the solution of the equation $\frac{\partial^2 V}{\partial r^2} + \frac{1}{r}\frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial z^2} = 0$ for the region $r \geq 0$, $z \geq 0$ satisfying the conditions

(i) $V \to 0$, as $z \to \infty$ and $r \to \infty$

(ii) $V = f(r)$ on $z = 0$, $r \geq 0$.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Reduce the PDE

$$y^2 \frac{\partial^2 z}{\partial x^2} - 2xy \frac{\partial^2 z}{\partial x \, \partial y} + x^2 \frac{\partial^2 z}{\partial y^2} = \frac{y^2}{x}\frac{\partial z}{\partial x} + \frac{x^2}{y}\frac{\partial z}{\partial y}$$ to canonical form the hence solve it.

17. Derive the interior Neumann problem for a circle.

18. Find the solution of diffusion equation in cylindrical co-ordinates.

19. Derive D'Alembert solution of one-dimensional wave equation.

20. Determine the solution for the displacement of an infinite string governed by $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, $-\infty < x < \infty$ subject to the boundary conditions $u(x, 0) = f(x)$, $-\infty < x < \infty$ and $\frac{\partial u}{\partial t}(x, 0) = 0$.

(For the candidates admitted from 2012–2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Second Semester

Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — ($10 \times 2 = 20$ marks)

Answer ALL questions.

1. Define complementary function.

2. Define quasi-linear PDE of second order.

3. Write Laplace equation in cylindrical coordinates.

4. State the interior Dirichlet problem for a circle.

5. State Neumann condition.

6. Write the diffusion equation in spherical co-ordinates.

7. Write the wave equation in cylindrical coordinates.

8. State Duhamel principle for wave equation.

9. State the Cauchy integral formula.

10. Write the Laplace transform of e^{at}.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If $u_1, u_2, \ldots u_n$ are solutions of the homogeneous linear partial differential equation $F(D, D')z = 0$, then show that $\sum_{r=1}^{n} C_r u_r$, where C_r' are arbitrary constants, is also a solution.

Or

(b) Solve the equation

$$\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} - \frac{\partial^3 z}{\partial x\, \partial y^2} + 2\frac{\partial^3 z}{\partial y^3} = e^{x+y}.$$

12. (a) Show that two dimensional Laplace equation $\nabla_1^2 V = 0$, in the plane polar coordinates r and θ has the solution of the for $\left(Ar^n + Br^{-n}\right)e^{\pm in\theta}$, where A, B and n are constants. Determine V if it satisfies $\nabla_1^2 V = 0$ in the region $0 \le r \le a$, $0 \le \theta \le 2\pi$. and

(i) V remains finite as $r \to 0$

(ii) $V = \sum_n C_n \cos(n\theta)$, on $r = a$.

Or

(b) Derive the Laplace equation.

13. (a) Derive the solution of one dimensional heat conduction equation in separation of variable method.

Or

(b) Explain the various types of boundary conditions for heat conduction equation.

14. (a) Determine the periodic solution of one dimensional wave equation in Spherical polar co-ordinates.

Or

(b) Obtain the solution of the radio equation $\frac{\partial^2 v}{\partial x^2} = LC\frac{\partial^2 v}{\partial t^2}$ appropriate to the case when a periodic e.m.f. $V_0 \cos pt$ is applied at the end $x = 0$ of the line.

15. (a) Using the Laplace transform method, solve the initial boundary value problem $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \cos wt$, $0 \le x \le \infty$, $0 \le t \le \infty$ subject to the initial and boundary conditions $u(0, t) = 0$, u is bounded as $x \to \infty$ $\frac{\partial u}{\partial t}(x, 0) = u(x, 0) = 0$.

Or