SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let p_k denote the k^{th} prime number, then if $s > 1$ we have $\zeta(s) = \sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{k=1}^{\infty} \frac{1}{1 - p_k^{-s}}$. Then show that the product converges absolutely.

17. Let Ω be the set of all invertible linear operator on R^n, the show that

    (a) If $A \in \Omega$, $B \in L(R^n)$ and $\|B - A\| \cdot \|A^{-1}\| < 1$, then $B \in \Omega$.

    (b) Ω is an open subset of $L(R^n)$ and the mapping $A \to A^{-1}$ is continuous on Ω.

18. If $[A]$ and $[B]$ are $n \times n$ matrices then prove that $\det([B][A]) = \det[B]\det[A]$.

19. Suppose F is a C' mapping of an open set $E \subset R^n$ into $\mathbb{R}^n$, $0 \in E$, $F(0) = 0$ and $F'(0)$ is invertible then prove that there is a neighborhood of 0 in R^n in which a representation $F(x) = B_1 \ldots B_{n-1} G_n \circ \ldots \circ G_1(x)$ is valid.

20. If ψ is a k-chain of class C'' in an open set $V \subset R^m$ and if ω is a $(k-1)$ form of class C' in V, then prove that $\int_{\psi} d\omega = \int_{\partial\psi} d\omega$.

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S.No. 232

12PMA06

(For the candidates admitted from 2012–2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Second Semester

Mathematics

ADVANCED REAL ANALYSIS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State Cauchy condition for product.

2. Define Cesarosummability.

3. Define basis of X.

4. Define continuously differentiable.

5. Define rank of matrix A.

6. Define null space.

7. Define basic k-forms,

8. Define k- surface in E.

9. Define standard simplex.

10. When we say a set is exact.

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) If $\lim_{p,q\to\infty} f(p,q) = a$. For each fixed p assume that the limit $\lim_{p,q\to\infty} f(p,q)$ exists then prove that the limit $\lim_{p,q\to\infty}\left(\lim_{q\to\infty} f(p,q)\right)$ also exists and has the value a.

Or

(b) If $a_n \geq 0$ then prove that the product $\prod(1-a_n)$ converges if and only if the series $\sum a_n$ converges.

12. (a) If $\bar{f}$ maps a convex open set $E \subset R^n$ into R^m, $\bar{f}$ is differentiable in E, and there is a real number M such that $\|\bar{f}'(x)\| \leq M$ for every $x \in R$. Then prove that $|\bar{f}(b) - \bar{f}(a)| \leq M|b-a|$ for all $a \in E$, $b \in E$.

Or

(b) If X is a complete metric space, and if φ is a contraction of X into X then show that there exists one and only one $x \in X$ such that $\varphi(x) = x$.

13. (a) Prove that a linear operator A on R^n is invertible if and only if $\det[A] \neq 0$. If $[A]$ and $[B]$ are n by n matrices then prove that $\det([B][A]) = \det[B]\det[A]$.

Or

(b) If f is defined in an open set $E \subset R^2$ and that $D_1 f$ and $D_2 f$ exists at every point of E and $D_{21} f$ is continuous at some point $(a,b) \in E$. Then $D_{12} f$ exists at $(a,b) \in E$. Then show that $D_{12} f$ exists at (a,b) and $(D_{12} f)(a,b) = (D_{21} f)(a,b)$.

14. (a) Suppose $\omega = \sum_1 b_1(x)\,dx_1$ is the standard representation of a k-form ω in an open set $E \subset R^n$. If $\omega = 0$ in E, then show that $b_1(x) = 0$ for every increasing k-index I and for every $x \in E$.

Or

(b) If ω is of class C' in E then show that $d^2\omega = 0$.

15. (a) State and prove Stoke's formula.

Or

(b) State and prove Green's theorem.