SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. Let $M' \xrightarrow{u} M \xrightarrow{v} M'' \to 0$ be a sequence of A-modules and homorphisms then show that the sequence $M' \xrightarrow{u} M \xrightarrow{v} M'' \to 0$ is exact $\Leftrightarrow$ for all A-modules N, the sequence $0 \to Hom\,(M'', N) \xrightarrow{\bar{v}} Hom\ (M, N) \xrightarrow{\bar{u}} Hom\,(M', N)$ is exact.

17. For any A-module M, then show that the following statements are equivalent :

(a) M is a flat A - module

(b) $M_{\wp}$ is flat $A_{\wp}$ - module for each prime ideal $\wp$.

(c) M_m is a flat A_m - module for each maximal idea m.

18. Let $A \subseteq B$ be integral domains, B integral over A. Then show that B is a field if and only if A is a field.

19. State and prove Hilbert's Basis theorem.

20. State and prove structure theorem for Artin rings.

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S.No. 231 **12PMA05**

(For the candidates admitted from 2012-2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Second Semester

Mathematics

ADVANCED ALGEBRA

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define A-module homorphism.

2. Define quotient ring.

3. Define tensor product.

4. State local property.

5. Define primary ideal in ring.

6. State Going-up theorem.

7. Define Artinian.

8. Define Noetherian ring.

9. Define Artin ring.

10. Define fractional ideal.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Show that every ring $A \neq 0$ has at least one maximal ideal.

Or

(b) If a, b be ideals in a ring A such that $r(a), r(b)$ are coprime then show that a,b are coprime.

12. (a) Let $x_1 \in M, y_1 \in N$ be such that $\sum x_1 \oplus y_1 = 0$ in $M \otimes N$. Then show that there exist finitely generated submodules M_0 of M and N_0 of N such that $\sum x_1 \otimes y_1 = 0$ in $M_0 \otimes N_0$.

Or

(b) Let M be an A-module. Then show that the following are equivalent :

(i) $M = 0$

(ii) $M_{\wp} = 0$ for all prime ideals $\wp$ of A.

(iii) $M_m = 0$ for all maximal ideals of A.

13. (a) State and prove Going — down theorem.

Or

(b) Prove that B is a local ring and $m = Ker(g)$ is its maximal ideal.

14. (a) Prove that in a Noetherian ring A every ideal is a finite intersection of irreducible ideals.

Or

(b) Prove that in a Noetherian ring every irreducible ideal is primary.

15. (a) Prove that in an Artin ring has only a finite number of maximal ideals.

Or

(b) Let A be an integral domain. Then prove that A is a Dedekind domain $\Leftrightarrow$ every non-zero fractional ideal of A is invertible.