SECTION C — (3 × 10 = 30 marks)

Answer any THREE questions.

16. State and prove Existence theorem.

17. Let b be continuous on an interval I then prove that every solution ψ of $L(y) = b(x)$ on I can be written as $\psi = \psi_p + c_1\phi_1 + c_2\phi_2$ where ψ_p is a solution of $L(y) = 0$ and $c_1, c_2, \ldots c_n$ are constants. A particular solution ψ_p is given by $\psi_p(x) = \int_{x_0}^{x} \frac{[\phi_1(t)\phi_2(x) - \phi_1(x)\phi_2(t)]\,b(t)}{W(\phi_1, \phi_2)(t)}\,dt$. Conversely every such ψ is a solution of $L(y) = b(x)$.

18. If $\phi_1, \cdots \phi_n$ are n solutions of $L(y) = 0$ on an interval I then show that they are linearly independent there if, and only if, $W(\phi_1, \cdots, \phi_n)(x) \neq 0$ for all x in I.

19. Derive the general solution of Bessel equation $x^2y'' + xy' + (x^2 - a^2)y = 0$.

20. Let M, N be two real-valued functions which have continuous first partial derivatives on some rectangle $R : |x - x_0| \leq a$, $|y - y_0| \leq b$. Then prove that the equation $M(x, y) + N(x, y)y' = 0$ is exact in R if, and only if, $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ in R.

12PMA04

(For the candidates admitted from 2012 – 2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

First Semester

Mathematics

ORDINARY DIFFERENTIAL EQUATIONS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Find all solutions of $y'' + y' - 2y = 0$.

2. Show that function $\phi_1(x) = x^2$, $\phi_2(x) = 5x^2$ is linear dependent.

3. Write the characteristic polynomial of $L(y) = y''' - 3r_1y'' + 3r_1^2y' - r_1^3y$.

4. Write the characteristic polynomial of an Annihilator for the following function :

   (a) e^{ax} and

   (b) $x^k e^{ax}$.

5. Define singular point.

6. When we say the solution of the equation $L(y)=0$ is linearly independent and linearly dependent.

7. Write the Bessel function of order $\alpha=0$ of the first kind.

8. Show that $P_1(x)=x$.

9. Define Lipschitz condition.

10. Is this equation $(x^2+xy)\,dx+xy\,dy=0$ exact?

SECTION B — (5 × 5 = 25 marks)

Answer ALL questions.

11. (a) Prove that two solutions ϕ_1,ϕ_2 of $L(y)=0$ are linearly independent on an interval I if, and only if, $W(\phi_1,\phi_2)\ (x)\neq 0$ for all x in I.

Or

(b) Find the solution of the initial value problem $y''-2y'-3y=0$, $y(0)=0$, $y'(0)=1$.

12. (a) Find all solution of $y''-y'-2y=e^{-x}$.

Or

(b) Find all real-valued solution $y^{(4)}+y=0$.

13. (a) One solution of $y''-\frac{2}{x^2}y=0$, for $(0<x<\infty)$ is $\phi_1(x)=x^2$. Find the basis of the for the solution.

Or

(b) Show that there exist n linearly independent solutions of $L(y)=0$ on I.

14. (a) Prove that $P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}(x^2-1)^n$ is the n^{th} – Legendre polynomial.

Or

(b) Show that $x^{1/2}J_{1/2}(x)=\frac{\sqrt{2}}{\Gamma\left(\frac{1}{2}\right)}\sin x$.

15. (a) Consider the initial value problem $y'=xy$, $y(0)=1$ then compute the solution using method of successive approximation.

Or

(b) Give an example of a function which satisfies Lipschitz condition and another function which does not satisfy the Lipschitz condition.