18. State and prove Brachistochorne problem.

19. State and prove Jacobi's theorem.

20. Show that the transformation is canonical $Q = \frac{1}{2}\left(q^2 + p^2\right), P = -\tan\frac{q}{p}$. If $H = \frac{1}{2}\left(q^2 + p^2\right)$ then find the new Hamilton.

S.No. 227

12PMA03

(For the candidates admitted from 2012–2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

First Semester

Mathematics

MECHANICS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. State Konig' s Theorem.

2. Define workless constraint.

3. State Kepler problem.

4. Define conservative system.

5. State principle's of least action.

6. Write the Jacobian form of principle's of least action.

7. State Stackel's theorem.

8. Write the Hamilton Jacobi equation.

9. Define momentum transformation.

10. Define Lagrange's brackets.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Derive the Lagrangian form of D'Alembert's principle.

Or

(b) A particle of mass m is suspended by a mass less wire of length $r = a + b\cos\omega t (a > b > 0)$ to form a spherical pendulum. Find the equation of motion.

12. (a) State and the Jacobi integrals have the unit of energy.

Or

(b) Derive the standard form of Lagrange's equation for a holonomic equation.

13. (a) Derive the Hamilton's equation.

Or

(b) Find the stationary value of the function $f = z$ subject to the constraints $\varphi_1 = x^2 + y^2 + z^2 - 4 = 0$; $\varphi_2 = xy - 1 = 0$

14. (a) Explain the Pfaffian differential form

Or

(b) Derive the modified-Hamilton's Jacobi equation.

15. (a) Show that Rheonomic transformation $Q = \sqrt{2q}\, e^t \cos p, P = \sqrt{2q}\, e^{-t} \sin p$ is canonical.

Or

(b) State and prove Jacobi's identity property.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. Derive the rotational kinetic energy in the form of $T_{rot} = \frac{1}{2}\omega^T I\omega$.

17. Discuss Routhian function and prove that $\frac{d}{dt}\left(\frac{\partial R}{\partial q_i}\right) = \frac{\partial R}{\partial q_i} = 0$.