17. Suppose f and g are defined on [a, b] and are differentiable at a point $x \in [a, b]$. Then show that $f+g$, fg and f/g are differentiable at x, and

(a) $(f+g)'(x) = f'(x) + g'(x)$;

(b) $(fg)'(x) = f'(x)g(x) + f(x)g'(x)$;

(c) $\left(\frac{f}{g}\right)'(x) = \frac{g(x)\, f'(x) - g'(x) f(x)}{g^2(x)}$, $g(x) \neq 0$.

18. If γ' continuous on $[a, b]$, then show that γ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt$.

19. Suppose $f_n \to f$ uniformly on a set E in a metric space. Let x be a limit point of E, and suppose that $\lim_{t \to x} f_n(t) = A_n (n = 1, 2, 3, ...)$. Then prove that $\{A_n\}$ converges, and $\lim_{t \to x} f(t) = \lim_{n \to \infty} A_n$.

20. State and prove Parseval's theorem.

S.No. 226 **12PMA02**

(For the candidates admitted from 2012 – 2013 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

First Semester

Mathematics

REAL ANALYSIS

Time : Three hours Maximum : 75 marks

SECTION A — (10 × 2 = 20 marks)

Answer ALL questions.

1. Define open cover.

2. Define uniform continuous.

3. Define Taylor's theorem.

4. Define local maximum.

5. Write equation of Riemann-Stieltjes integrals.

6. Define refinement.

7. Define pointwise bounded.

8. Define uniformly bounded.

9. Define analytic function.

10. Define the gamma function.

SECTION B — ($5 \times 5 = 25$ marks)

Answer ALL questions.

11. (a) Show that compact subsets of metric spaces are closed.

Or

(b) If f is a continuous mapping of a compact metric space X into a metric space Y then prove that f is uniformly continuous on X.

12. (a) If f is defined on $[a, b]$ and it has a local maximum at a point $x \in (a, b)$ and if $f'(x)$ exists then show that $f'(x) = 0$.

Or

(b) State and prove Taylor's theorem.

13. (a) Show that if $f \in \mathfrak{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such that $U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon$

Or

(b) Prove that $\int_{-a}^{b} f\,dx \leq \int_{a}^{-b} f\,dx$.

14. (a) State and prove Stone-Weierstrass theorem.

Or

(b) If K is a compact metric space, if $f_n \in \ell(K)$ $n = 1, 2, 3, \ldots..,$ and if $\{f_n\}$ converges uniformly on K, then prove that $\{f_n\}$ is equicontinuous on K.

15. (a) Prove that $\beta(x, y) = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$.

Or

(b) If f is continuous (with period 2π) and if $\varepsilon > 0$, then show that there is a trigonometric polynomial P such that $|P(x) - f(x)| < \varepsilon$ for all real x.

SECTION C — ($3 \times 10 = 30$ marks)

Answer any THREE questions.

16. If f is a continuous mapping of a compact metric space X into a metric space Y then prove that $f(X)$ is compact.