(6 pages)

S.No. 220

08PMA09/08PMY15

(For the candidates admitted from 2008–2009 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Third & Fourth Semester

Mathematics

MEASURE THEORY AND INTEGRATION

(Common for Maths (C.A.))

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 5 = 25 marks)

Answer ALL questions.

1. (a) Let $\{A_n\}$ be a countable collection of sets of real number then show that $m^*(\bigcup A_n) \leq \sum m^* A_n$.

Or

(b) Show that the interval (a, ∞) is measurable.

2. (a) If f and g are bounded measurable function defined on a set E of finite measure then show that $\int_E (af + bg) = a\int_E f + b\int_E g$.

Or

(b) State and prove Lebesgue convergence theorem.

3. (a) Write the formula for :

(i) $D^+ f(x)$

(ii) $D^- f(x)$

(iii) $D_+ f(x)$ and

(iv) $D_- f(x)$.

Or

(b) State and prove Vitali lemma.

4. (a) State and prove monotone convergence theorem.

Or

(b) If $E_i \varepsilon \mathrm{B}$, $\mu E_1 < \infty$ and $E_i \supset E_{i+1}$ then prove that $\mu\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} \mu E_n$.

5. (a) Let x be a point of X and E a set in $\Re_{\sigma\delta}$. Then prove that E_x is a measurable subset of Y.

Or

(b) Show that the set function μ^* is an outer measure.

SECTION B — (5 × 10 = 50 marks)

Answer ALL questions.

6. (a) If $\langle E_n \rangle$ be an infinite decreasing sequence of measurable sets, that is, a sequence with $E_{n+1} \subset E_n$ for each n. If mE_1 be finite then prove that $m\left(\bigcap_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} mE_n$.

Or

(b) Let c be a constant and f and g two measurable real-valued functions defined on the same domain. Then prove that the functions $f+c, cf, f+g, g-f$ and fg are also measurable.

7. (a) State and prove Bounded convergence theorem.

Or

(b) Let f and g be integrable over E. Then prove that :

(i) The function cf is integrable over E, and $\int_E cf = c\int_E f$.

(ii) The function $f+g$ is integrable over E, and $\int_E f+g = \int_E f + \int_E g$.

(iii) If $f \le g$ always everywhere, then $\int_E f \le \int_E g$.

(iv) If A and B are disjoint measurable sets contained in E, then $\int_{A\cup B} f = \int_A f + \int_B f$.

8. (a) Let f be a integrable function on $[a, b]$ and suppose that $F(x)=F(a)+\int_a^x f(t)dt$ then show that $F'(x)=f(x)$ for all x in $[a, b]$.

Or

(b) Let f be an increasing real-valued function on the interval $[a, b]$. Then prove that f is differentiable almost everywhere. The derivative f' is measurable, and $\int_a^b f'(x)dx \leq f(b)-f(a)$.

9. (a) State and prove Fatou's lemma.

Or

(b) State and prove State Radon-Nikodym theorem.

10. (a) State and prove Fubini theorem.

Or

(b) State and prove Carathedory theorem.
