9. (a) State and prove Uniform continuity theorem.

Or

(b) Prove that every compact subspace of Hausdorff space is closed.

10. (a) State and prove Urysohn metrization theorem.

Or

(b) State and prove Tietze extension theorem.

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08PMA08/ 08PMY10

(For the candidates admitted from 2008-2009 onwards)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2017.

Third Semester

Mathematics & Maths (CA)

TOPOLOGY

(Common for M.Sc. Maths (CA))

Time : Three hours Maximum : 75 marks

SECTION A — (5 × 5 = 25 marks)

Answer ALL questions.

1. (a) Show that every finite point set in a Hausdorff space X is closed.

Or

(b) If X is a set and if $\mathfrak{B}$ is a basis for a topology τ on X. Then prove that τ equals the collection of all unions of elements of $\mathfrak{B}$.

2. (a). Define : (i) Product topology and (ii) Metric topology.

Or

(b) State and prove Pasting Lemma.

3. (a) Prove that the unit ball B^n in $\Re^n$ is path connected.

Or

(b) Show that a space X is locally connected if and only if for every open set U of X, each component of U is open in X.

4. (a) State and prove the Lebesgue number lemma.

Or

(b) Prove that the image of a compact space under a continuous map is compact.

5. (a) Suppose that X has a countable basis. Then prove that every open covering of X contains a countable sub-collection covering X.

Or

(b) Show that every compact Hausdorff space is normal.

SECTION B — (5 × 10 = 50 marks)

Answer ALL questions.

6. (a) Let A be a subset of the topological space X and A' be the set of all limit points of A then show that $\overline{A} = A \cup A'$.

Or

(b) Let X be a topological space. Suppose that $\mathcal{C}$ is a collection of open sets of X such that for each open set U of X and each x in U, there is an element C of $\mathcal{C}$ such that $x \in C \subset U$. Then show that $\mathcal{C}$ is a basis for the topology of X.

7. (a) Prove that the topologies on $\Re^n$ induced by the Euclidean metric d and the square metric ρ are the same as the product topology on $\Re^n$.

Or

(b) State and prove uniform limit theorem.

8. (a) Show that the union of a collection of connected subspace of X that have a common point is connected.

Or

(b) State and prove intermediate value theorem.